Surface Tracing in 3D

Let me know what you think

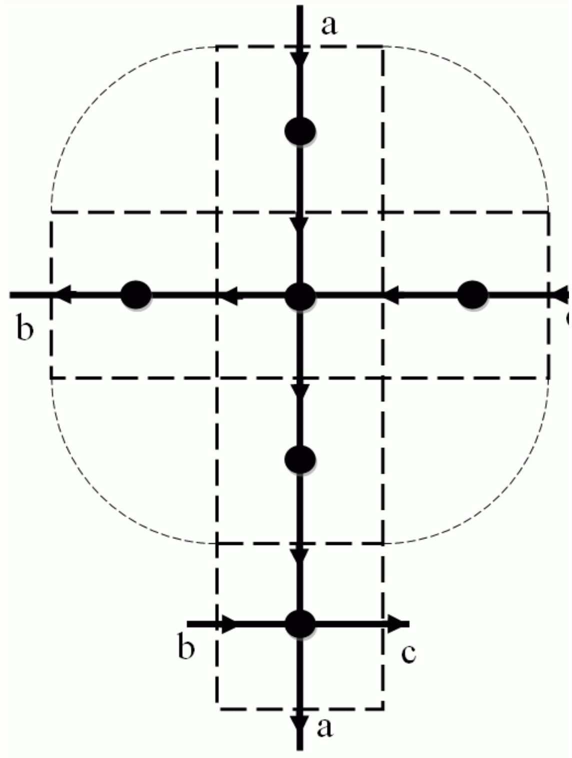
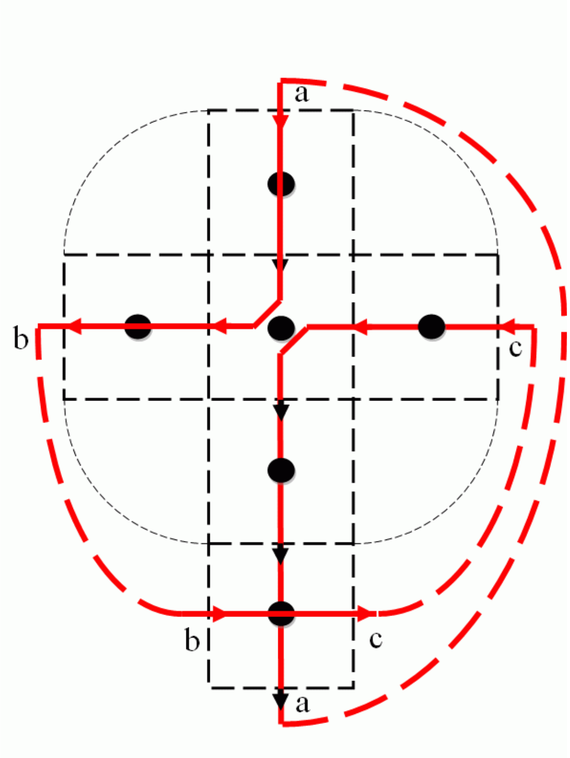
The author has developed three algorithms for economical and exact encoding of surfaces in 3D spaces. He has compared them with the algorithm known as "Depth-First Search" of a graph.

Depth-First Search

It is well-known that the set of the facets of a surface can be considered as a graph: facets are the vertices and any two adjacent facets are connected by an edge of the graph. The aim of the algorithm is to put all facets of the surface into a list. If a facet is adjacent to that previously put into the list, then only the differences of coordinates are saved in the list. Otherwise coordinates are saved. The algorithm starts with an arbitrary facet, puts it into the stack ("last in first out") and starts a while-loop which is running while the stack is not empty. In the loop a facet *F* is being popped form the stack. If it is not labeled as being already in the list then it is put into the list and labeled.. Then all facets adjacent to *F* are put into the stack. This algorithm is rather simple and fast, but it is not economical: It needs on an average 2.5 bytes per facet even if sequences of adjacent facets are coded by differences of coordinates. The sequences are mostly too short.

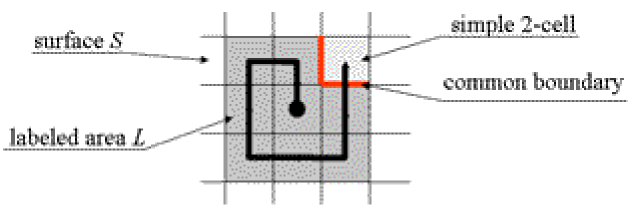
Euler Circuit

The author has developed an algorithm which finds the Euler circuit of the adjacency graph of the facets. The algorithm uses a directed graph (digraph) with the aim to make the number of edges as small as possible. The Euler circuit is a closed sequence of facets and the 0- or 1-cells in between containing each 0- or 1-cell only once (a facet can be contained twice). The Euler circuit can be encoded very economically by the differences of the coordinates of two adjacent facets. Experiments have showed that this code needs on an average about 0.75 bytes per facet.

Digraph of the adjacencies in the surface of a small cube	Euler circuit of the surface of a small cube
	

Spiral Tracing

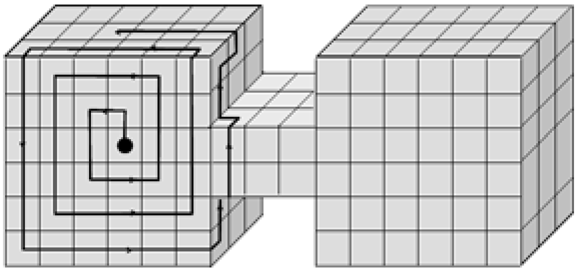
This algorithm takes an arbitrary facet of the surface *S* as a starting one and labels its closure. This is the "germ-cell" of the set *L* of the labeled facets. The algorithm traces the opening boundary (see lecture "Introduction") of *L* and labels all "simple" facets. A facet is simple if the intersection of its boundary with the boundary of *L* is not empty and connected while the intersection of its boundary with the complement of *L* is also not empty. The trace looks like a spiral. This method is less efficient than that of Euler circuit: it needs between 1.0 and 2.0 bytes per facet depending on the number of tunnels in the surface. It is, however, interesting since it defines the genus (number of tunnels) of the surface.



The idea of the tracing:

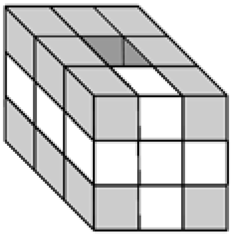
The set *L* of labeled simple facets remains always topologically equivalent to a disk (a 2-ball).

Example of tracing

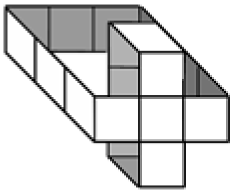


Recognition of the Genus of the Surface

The set of simple facets (grey) composes a topological disk (*2*-ball)



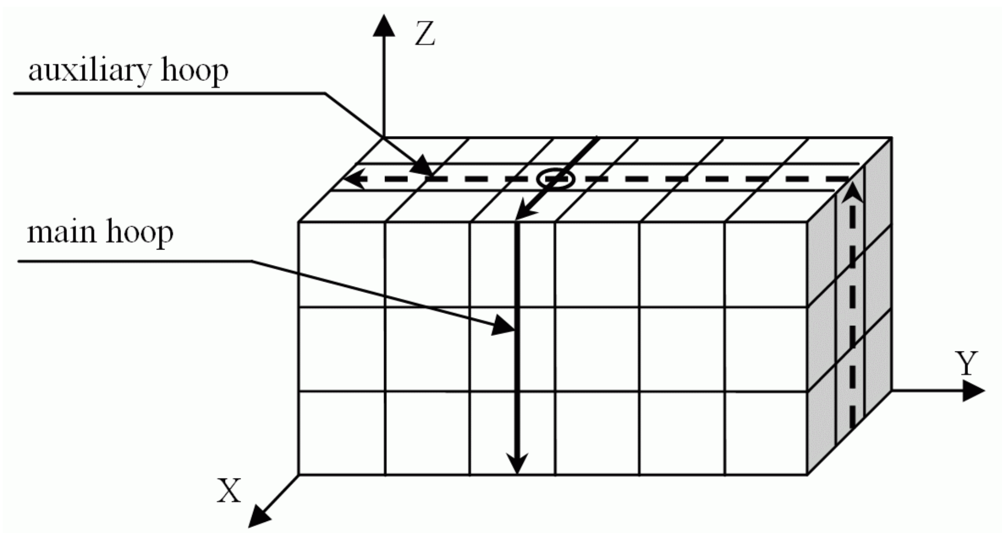
The set of remaining non-simple cells (white) carries information about the genus.



Economical Hoop Code

This is the most economical method. It finds and traces facets lying in the boundary of a two-dimensional slice of the body (solid line in the figure below). This boundary is called a "hoop". Another "auxiliary hoop" (dashed line) is used for finding the starting facets of the main hoops.

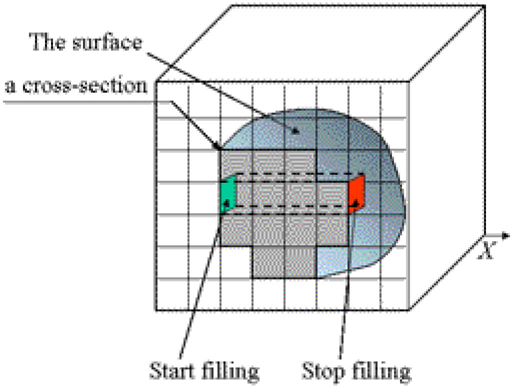
It is sufficient to save the coordinates of a single starting facet of a hoop. For all other facets only the differences of their coordinates are encoded. The efficiency of this method is between 0.21 and 0.5 byte per facet for bodies whose surface has no singularities. However, for bodies with singular surfaces the efficiency can be rather bad. This is the main drawback of this method.



Filling the Interiors of Surfaces in 3D

The algorithm is similar to that presented in Lecture 2 for filling interiors of curves in 2D. It is necessary to label the facets (2-cells) of the given surface whose normals are parallel (or anti-parallel) to one of the coordinate axes. Then the algorithms scans all rows of the grid, that are parallel to the chosen axis, and counts the labeled facets. The filling begins at each odd count and ends at each even count.

The surface



The pseudo code

Choose a coordinate axis of the Cartesian space, e.g. the *X*-axis.
Label all (n−1)-cells of *M* whose normal is parallel to *X*.

```
for ( each row R parallel to X )
{ fill=FALSE;
  for each n-cell C in the row R
  { if the 1st (n-1)-side of C is labeled
      fill = NOT fill;
    if ( fill==TRUE ) C = foreground;
    else              C = background;
  }
}
```

If you are interested in theoretical details then read the lection "Axiomatic Digital Topology" below.

References

[1] V. Kovalevsky, *A Topological Method of Surface Representation*, In: Bertrand, G. et all (eds.), Lecture Notes in Computer Science, vol. 1568, Springer 1999, pp. 118-135.

[2] V. Kovalevsky: *Geometry of Locally Finite Spaces*, Monograph, Berlin 2008.